

William Shanks 707 digits

January 1, 2012

The following value of PI is from the "Ripley's believe it or not" 1940 edition. William Shanks, who in 1873 he took parts of 20 years to calculate an additional 100 digits, to finish with 707 digits of PI by hand, although due to a mistakes only the first 527 were correct. Due to known printing errors in the 1873 the typo errors added to the calculated errors. The below was the first recorded value I could find, which also had the errors in the first 527 digits, see the under lined three digits. This number was engraved in the PI room in Paris, and banners in classrooms all over the world. Before 1945 it was suspected there was a problems with the number, as each digit was not about 10%, there were only 53 7's. See page 8 for a listing of these numbers and if others had done the same they may not have questioned it. Curiosity it remained the standard until 1945 when D. F. Ferguson discovered that Shanks had made an error in the 528th digit and beyond. Ferguson and Dr. John W. Wrench Jr. set and beat their own records on 1945 620 digits, 1946 710 digits and 1947 808 digits.

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3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70679
82148 08651 32823 06647 09384 46095 50582 23172 53594 08128
48111 74502 84102 70193 85211 05559 64462 29489 54930 38196
44288 10975 66593 34461 28475 64823 37867 83165 27120 19091
45648 56692 34603 48610 45432 66482 13393 60726 02491 41273
72458 70066 06315 58817 48815 20920 96282 92540 91715 36436
78925 90360 01133 05305 48820 46652 13841 46951 94151 16094
33057 27036 57595 91953 09218 61173 81932 61179 31051 18548
07446 23798 34749 56735 18857 52724 89122 79381 83011 94912
98336 73362 44065 66430 86021 39501 60924 48077 23094 36285
53096 62027 55693 97986 95022 24749 96206 07497 03041 23668
86199 51100 89202 38377 02131 41694 11902 98858 25446 81639
79990 46597 00081 70029 63123 77381 34208 41307 91451 18398
05709 85+
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The first correct 709 digits of PI, the under lined portion was the part in error.

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3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70679
82148 08651 32823 06647 09384 46095 50582 23172 53594 08128
48111 74502 84102 70193 85211 05559 64462 29489 54930 38196
44288 10975 66593 34461 28475 64823 37867 83165 27120 19091
45648 56692 34603 48610 45432 66482 13393 60726 02491 41273
72458 70066 06315 58817 48815 20920 96282 92540 91715 36436
78925 90360 01133 05305 48820 46652 13841 46951 94151 16094
33057 27036 57595 91953 09218 61173 81932 61179 31051 18548
07446 23799 62749 56735 18857 52724 89122 79381 83011 94912
98336 73362 44065 66430 86021 39494 63952 24737 19070 21798
60943 70277 05392 17176 29317 67523 84674 81846 76694 05132
00056 81271 45263 56082 77857 71342 75778 96091 73637 17872
14684 40901 22495 34301 46549 58537 10507 92279 68925 89235
42019 9561
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For those who cannot find their own copy of the 1940 “Ripley's believe it or not”. I have included a scanned image of my copy for your own enjoyment. My copy of the book came with the passing of my Grandmother in 1964. I do not know why my father thought this book would be so enjoyed by me over the years but it has been. At the time I knew that it had errors but I did not know where the error was and that there was an extra three digit error in the beginning, even though it was known since 1873 which was before it came out in print. I was able to find the other web pages by searching for a string of error numbers in quote marks, see page 5. The 1873 Proceedings of the Royal Society of London, Volume 21 page 319 which was known to also to have typo errors, this has the same numbers printed in “Ripley's believe it or not” 1940 edition page 156, which did including the three digits typo error even though it was known at the time. The three digit typo error should have been corrected as it had been known since the original printing in 1873. As you may have noted there is a spacing error with six digits in one group and latter the group with four digits to correct the error. This whole book is available on the web.

“PI”

The letter Π (pi) indicates the incommensurable relationship between the diameter of any circle and its circumference. The number is usually given as 3.14159+. Here is the value worked out to 707 decimal places. The figures are not to be found in many places and they may be desired as a curiosity by some amateur mathematicians:

3.14159 26535 89793 23846 26433 83279 50288 41971
 69399 37510 58209 74944 59230 78164 06286 20899
 86280 34825 34211 70679 82148 08651 32823 06647
 09384 46095 50582 23172 53594 08128 48111 74502
 84102 70193 85211 05559 64462 29489 54930 38196
 44288 10975 66593 34461 28475 64823 37867 83165
 27120 19091 45648 56692 34603 48610 45432 66482
 13393 60726 02491 41273 72458 70066 06315 58817
 48815 20920 96282 92540 91715 36436 78925 90360
 01133 053054 88204 66521 38414 69519 41511 60943
 30572 70365 75959 19530 92186 11738 19326 11793
 10511 85480 74462 3798 34749 56735 18857 52724
 89122 79381 83011 94912 98336 73362 44193 66430
 86021 39501 60924 48077 23094 36285 53096 62027
 55693 97986 95022 24749 96206 07497 03041 23668
 86199 51100 89202 38377 02131 41694 11902 98858
 25446 81639 79990 46597 00081 70029 63123 77381
 34208 41307 91451 18398 05709 85+.

The 1873 Proceedings of the Royal Society of London, Volume 21 page 319 which was known to have typo errors, it also had the two sub terms William Shanks used to arrive at PI. You can see both terms had an error which was introduced into the final answer. Take 16 times the first value less 4 times the second value and you have PI. There was a typo error in the 75th digit, in the second line; the 7 should be an 8, the number block 69977 should read 69978. the location of these numbers can be found at “archive.org/stream/philtrans00902295/00902295#page/n1/mode/2up”.

ATN(1/5) = 0.

19739	55598	49880	75837	00497	65194	79029	34475	85103	78785	21015	17688
94024	10339	69977	24378	57326	97828	03728	80441	12628	11807	36913	60104
45647	98867	94239	35574	75654	95216	30327	00522	10747	00156	45015	56006
12861	85526	63325	73186	92806	64389	68061	89528	40582	59311	24251	61329
73139	93397	11323	35378	21796	08417	66483	10525	47303	96657	25650	48887
81553	09384	29057	93116	95934	19285	18063	64919	69751	94017	08560	94952
73686	73738	50840	08123	67856	15800	93298	22514	02324	66755	49211	02670
45743	78815	47483	90799	78985	02007	51136	96837	96139	22783	54193	25572
23284	13846	47744	13529	09705	46512	24383	02697	56051	83776	17781	64242
33783	03370	18192	64880	28277	68611	91509	85606	75901	21359	85563	63034
32100	56649	97826	76360	88711	52327	56610	84900	93773	38023	19504	70687
65729	38513	59243	19759	37947	36057	50636	20935	07853	2833		

ATN(1/239) = 0.

00418	40760	02074	72386	45382	14959	28545	27410	48065	30763	19508	27019
61288	71817	78341	42289	32737	82605	81362	29094	54975	45066	64448	63756
05245	83947	89311	86505	89221	28833	09280	08462	71962	33077	33759	47634
60331	84734	14570	33198	60154	54814	80599	24498	30211	46039	12539	49527
60779	68815	58881	27339	78533	46518	04574	25481	35867	46447	51979	10232
83097	70020	64652	82763	46532	96910	48183	86543	56078	91959	14512	32220
94463	68627	66155	20831	67964	26465	74655	11032	51034	35262	82445	12693
55670	49968	44452	47904	33177	28393	07086	31401	93869	51950	37058	64107
70855	85540	45223	55388	14237	67708	36515	69182	52702	00229	30895	44950
04358	54409	34496	44014	24187	24950	92283	86239	54553	33565	16494	21220
06852	38821	94006	45849	29313	23886	73467	64889	18731	81682	83021	21101
37897	11546	96191	84692	18237	33903	04682	04140	79985	6684		

In January 2012 I used the last four rows, 229 digits, of the above two numbers starting with 16 times the first group less 4 times the second group giving the last four rows of his PI value. The only error is in the third row with a 51111 not the printed value of 51100. This seems to be the first reference to this error; it would seem no one else has challenged his final calculation to produce his value of PI. The 11 units of error cannot be produced by a single digit error in the first term, numbers like 16, 12 or 4 can be understood not 11, and the last step has a series of 3's in the place of the error. This may be an error in arithmetic carry because it has never been corrected in any other publication. This error seems to have gone undetected for 139 years. On page 19 I have expanded ten digits around this error to see how could have been done.

89122	79381	83011	94912	98336	73362	44065	66430	86021	39501	60924	48077
23094	36285	53096	62027	55693	97986	95022	24749	96206	07497	03041	23668
86199	51111	89202	38377	02131	41694	11902	98858	25446	81639	79990	46597
00081	70029	63123	77381	34208	41307	91451	18398	05709	8592		

Here is one other source for the original values of PI to 707 digits. Note the three digits in the first 527 are correct and only one digit, the 565th digit is a 2 and should be a 3, the block is 55692 and should have been 55693.

matematikdunyasi.org/arsiv/makalelert/kimkorkar_pi.pdf

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3.14159 26535 89793 23846 26433 83276 50288 41971 69399 37510
 58209 74944 59230 78164 06286 20899 86280 34825 34211 70679
 82148 08651 32823 06647 09384 46095 50582 23172 53594 08128
 48111 74502 84102 70193 85211 05559 64462 29489 54930 38196
 44288 10975 66593 34461 28475 64823 37867 83165 27120 19091
 45648 56692 34603 48610 45432 66482 13393 60726 02491 41273
 72458 70066 06315 58817 48815 20920 96282 92540 91715 36436
 78925 90360 01133 05305 48820 46652 13841 46951 94151 16094
 33057 27036 57595 91953 09218 61173 81932 61179 31051 18548
 07446 23799 62749 56735 18857 52724 89122 79381 83011 94912
 98336 73362 44065 66430 86021 39501 60924 48077 23094 36285
53096 62027 55692 97986 95022 24749 96206 07497 03041 23668
86199 51100 89202 38377 02131 41694 11902 98858 25446 81639
79990 46597 00081 70029 63123 77381 34208 41307 91451 18398
05709 85+

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This is from another web location; it did correct the 3 digits typo error.

archive.org/stream/formulairedemat04peangoog/formulairedemat04peangoog_djvu.txt

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3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
 58209 74944 59230 78164 06286 20899 86280 34825 34211 70679
 82148 08651 32823 06647 09384 46095 50582 23172 53594 08128
 48111 74502 84102 70193 85211 05559 64462 29489 54930 38196
 44288 10975 66593 34461 28475 64823 37867 83165 27120 19091
 45648 56692 34603 48610 45432 66482 13393 60726 02491 41273
 72458 70066 06315 58817 48815 20920 96282 92540 91715 36436
 78925 90360 01133 05305 48820 46652 13841 46951 94151 16094
 33057 27036 57595 91953 09218 61173 81932 61179 31051 18548
 07446 23799 62749 56735 18857 52724 89122 79381 83011 94912
 98336 73362 44065 66430 86021 39501 60924 48077 23094 36285
53096 62027 55693 97986 95022 24749 96206 07497 03041 23668
86199 51100 89202 38377 02131 41694 11902 98858 25446 81639
79990 46597 00081 70029 63123 77381 34208 41307 91451 18398
05709 85+

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Another source on the web is “The Chronology of PI”, with the three digit error.

“books.google.com/books?id=QlbzjN_5pDoC&pg=PA282&lpg=PA282&dq=%22The+Chronology+of+PI%22&source=bl&ots=bLRDigUEy2&sig=_qLCJWfUSwmSRlssDTwaP25DLFQ&hl=en&sa=X&ei=VHQjT8HCDoiv0AGt1pT5CA&ved=0CCsQ6AEwATgK#v=onepage&q=%22The+Chronology+of+PI%22&f=true”

From a German book “Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht, Volume 26 page 263, I found this value” This value was retyped, as it had 45 digits per line and not the 60. They also made a table of the number of times each digit is used and 7’s came up short. The error in the first 527 digits was corrected while the last 180 have the same error. The way to find these places is to search for different 15 digits groups from the error digits like “41307 91451 18398” they must be place in quotes marks. Search for different group of error digits and get other locations.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399
 37510 58209 74944 59230 78164 06286 20899 86280 34825
 34211 70679 82148 08651 32823 06647 09384 46095 50582
 23172 53594 08128 48111 74502 84102 70193 85211 05559
 64462 29489 54930 38196 44288 10975 66593 34461 28475
 64823 37867 83165 27120 19091 45648 56692 34603 48610
 45432 66482 13393 60726 02491 41273 72458 70066 06315
 58817 48815 20920 96282 92540 91715 36436 78925 90360
 01133 05305 48820 46652 13841 46951 94151 16094 33057
 27036 57595 91953 09218 61173 81932 61179 31051 18548
 07446 23799 62749 56735 18857 52724 89122 79381 83011
 94912 98336 73362 44065 66430 86021 39501 60924 48077
 23094 36285 53096 62027 55693 97986 95022 24749 96206
 07497 03041 23668 86199 51100 89202 38377 02131 41694
 11902 98858 25446 81639 79990 46597 00081 70029 63123
 77381 34208 41307 91451 18398 05709 85+

On page 3 there are the two sub calculations that William Shanks used to produce the final answer for PI. The difference between his term and the correct value starting at the 531st digit is 0 43560 76458 etc there are 529 0’s before each value. The closest term is $(1/5)^{755} / 755$ which the same position digits are 0 25101 50578 etc. Each term will change by 1/25 the previous term, the one before is 6 29204 41131 etc and the one after is 0 01001 40749 etc. The second term the difference between his term and the correct value starting at the 591st digit is 04774 47473 etc. The closest term is $(1/239)^{247} / 247$ which has a value of 1 39002 14834 which is too large while the next term is 00002 41392 which is too small. In each number it is not missing a term that produced the error. If you want to see the detailed listing of each term needed to calculate PI look at the following web link. “engert.us/erwin/miscellaneous/PI.pdf”

As you can see there must be a calculation error that was carried through from some previous term. Many books have stated the error was produced by two missing terms I do not see this could be the correct solution to the problem I have never seen anyone say which terms. See more information on page 9 with a dropped digit. William Shanks did make an error in his calculation and at the same time he did set a new record beating the old 500 digit record with 527 digits. He was the last person to do the hand calculation of PI which most likely will not be broken. To set a new record a single person would have to do all the work by hand with no help from any other person or machine and prove it. Do not look down on his work emphasizing on his error rather recognize his accomplishment in the history of PI and other numbers. Yes there were people before 1945 that did memorize the 707 digits of PI just like today.

Three of the most important calculation of PI was as follows. Archimedes of Syracuse (287-212 BC), he obtained the approximation $223/71 < \pi < 22/7$, using a 96 side polygon. According to “files.eric.ed.gov/fulltext/ED175693.pdf” page 4 a 1967 history of PI, he also did $211872/67441 < \pi < 195882/62351$, this is from Heron of Alexandria (75-110), from a lost document of Archimedes. In 1844 Zacharias Dase a calculating prodigy in two months calculated correctly 200 decimals of PI in his head, without pen or paper. He used the slow converging relation $\pi = 4 * \text{ATN}(1/2) + 4 * \text{ATN}(1/5) + 4 * \text{ATN}(1/8)$. This record will most likely never be broken we do not seem to have people to do this work. William Shanks, who took many years over a 20+ year period to calculate PI to 709/707 digits in stages, due to mistakes only the first 527 digits were correct. William Shanks used the formula $\pi = 16 \text{ATN}(1/5) - 4 * \text{ATN}(1/239)$. This record would most likely never be broken. It would be hard to prove that the person did not use a computer printout to check his results. William Shanks was at the leading edge by working beyond the 441 and latter 500 known digits of the time. In 1945 D F Ferguson used a calculator to calculated PI using the formula $\pi = 12 \text{ATN}(1/4) + 4 * \text{ATN}(1/20) + 4 * \text{ATN}(1/1985)$, this was the start of machine doing the work. He broke his original record twice. This record is broken every few years. He found that his value disagreed with that of Shanks in the 528th place. Ferguson discovered that Shanks had omitted two terms which caused his error, see page 4 and 5.

Here are my calculations for the two sub terms that William Shanks should have produced with the total error in the value of PI would 105 units in the last digits. If all William Shanks did were 709 digits as in the two sub terms, like he did in the 607 with 609 calculations the last 3 digits could have been in error which would have been fine. To print this total calculation to 709 digits would take about 300 pages; this would not include all the long divisions. I am sure William Shanks did not write 60 digits per line as my program. In my hand calculation I did 50 digits across the width of a page, for the 100 digits I used 108 page and William may have used 5400 pages.

ATN(1/5) = +.
 19739 55598 49880 75837 00497 65194 79029 34475 85103 78785 21015 17688
 94024 10339 69978 24378 57326 97828 03728 80441 12628 11807 36913 60104
 45647 98867 94239 35574 75654 95216 30327 00522 10747 00156 45015 56006
 12861 85526 63325 73186 92806 64389 68061 89528 40582 59311 24251 61329
 73139 93397 11323 35378 21796 08417 66483 10525 47303 96657 25650 48887
 81553 09384 29057 93116 95934 19285 18063 64919 69751 94017 08560 94952
 73686 73738 50840 08123 67856 15800 93298 22514 02324 66755 49211 02670
 45743 78815 47483 90799 78985 02007 52236 96837 96139 22783 54193 25572
 23284 13846 47744 13529 09705 46512 24383 02697 56051 83775 74220 87783
 58531 52464 74933 09145 87633 82311 24903 32030 12680 51006 70223 31257
 50509 42448 46026 71622 54894 07922 61404 67995 06236 59692 82873 05828
 78720 53603 03457 07660 66681 37431 25662 67431 40899 2599

ATN(1/239) = +.
 00418 40760 02074 72386 45382 14959 28545 27410 48065 30763 19508 27019
 61288 71817 78341 42289 32737 82605 81362 29094 54975 45066 64448 63756
 05245 83947 89311 86505 89221 28833 09280 08462 71962 33077 33759 47634
 60331 84734 14570 33198 60154 54814 80599 24498 30211 46039 12539 49527
 60779 68815 58881 27339 78533 46518 04574 25481 35867 46447 51979 10232
 83097 70020 64652 82763 46532 96910 48183 86543 56078 91959 14512 32220
 94463 68627 66155 20831 67964 26465 74655 11032 51034 35262 82445 12693
 55670 49968 44452 47904 33177 28393 07086 31401 93869 51950 37058 64107
 70855 85540 45223 55388 14237 67708 36515 69182 52702 00229 30895 44950
 04358 54409 34496 44014 24187 24950 92283 86239 54553 33565 11719 73747
 02023 49475 97790 97469 50111 88854 76673 97957 31537 09303 27821 13089
 84258 30836 77190 91008 39098 51655 10419 22416 78092 0532

From the "Mathematical Tables and Other Aids to Computation" Vol. 2, No. 18 (Apr., 1947), pp. 245-248 Published by: American Mathematical Society page 245. Article Stable URL: <http://www.jstor.org/stable/2002296>

A New Approximation to π

A. Editorial Note: In MTAC v. 2 p. 143-145 we noted various formula which had been used for calculating π to many places of decimals. These included that of Machin (1706)

$$(1) \quad \pi = 16 \cdot \text{ATAN}(1/5) - 4 \cdot \text{ATAN}(1/239)$$

which as used by William Shanks (1812-1882) to compute π to 707D. The accuracy of this computation to 500D as verified by an independent calculation completed and published in 1854. No one appears to have checked the later figures until 1945, when Mr. D. F. Ferguson, now connected with the Department of Mathematics of the University of Manchester, undertook the task. As we have already noted he used the formula

$$(2) \quad \pi = 12 \cdot \text{ATAN}(1/4) + 4 \cdot \text{ATAN}(1/20) + 4 \cdot \text{ATAN}(1/1985)$$

given in Loney's Plane Trigonometry (1893). We published his new computation for π from the 526th through the 620th decimal place (p, 145). Mr. Ferguson gave an account of his work in Mathematical Gasetts v. 30 May 1946, p. 89-90, and recorded there his figures for π from the 521st to the 540th decimal place. Mr. Ferguson found that Shanks' approximations to π as incorrect beyond 527D. By November 1946 he had carried on his calculations of the value of π to 700D, and by January 1947 to 710D.

In December 1945 we suggested to Dr. John W. Wrench Jr. that he might take up the wholly independent computation of π by means of the Machin's formula (1). In April 1946 he reported that he in communication with Mr. Levi B. Smith of Talbotton, Georgia, who began his work on computing $\text{ATAN}(1/239)$ in November 1940 and had by February 1944 completed the work to 820D, through $1/(173 \cdot 239^{173})$. Then Dr. W. took up the actively the computation of $\text{ATAN}(1/5)$ so that his result might be combined with those of Mr. S. as in Machin's formula. He found the errors in work of Shanks, earlier pointed out by Mr. F., and described below.

Early in January 1947 Dr. W. sent to us his new Approximation to π to 808D (MATC, v. 2, April 1946, p. 69). The value found by Mr. F to 719D agrees with this. 808D of π printed here

Here are the last 10 digits of his 810 digits work which by the way were correct to all 810 digits, which are "50244 594(55)" the part in brackets were dropped to list only 808 digits. In my calculation using the same formula the last 10 digits were "50244 59528" the last three digits were in error.

This writing had to be done before 1949 with the advent of the computer. The date mentioned at the beginning of this article is January and printed in April 1947. The work done by Smith had reached the term with 413D of "0's", which was about 75% of the work for his contribution for the $\text{ATAN}(1/239)$. I believe the dates to be the most reliable date as it was a few years after the event, and the people involved were still alive. This was the first use of two different methods to check the value to be correct.

Here is the distribution of digits for PI. The first line starts with the number of digits tested followed by the count of each digit 0 through 9. The second line has the percent that each digit is found for 0 through 9. For the first two lines is a test of the first 100 digits of PI less the first 3. The number 0 is found 8 times which 8.00% of the total of 100 digits. In the seventh set which starts with the 700 the 0 digit is found 64 times and is 9.14% of the total 700 digits. By 2000 the numbers hugs the 10% value, 700 did better than 500.

This is the results for William Shanks value for the first 707 digits of PI. Note that there are only 53 7's in his value which is only 7.50%, not close enough to 10% this was noted shortly after it as published in 1873 and nothing was done about it. Check the breakdown of the first 707 digits from Williams Shanks and the first 1000 correct digits, they checked in blocks of 100 might have felt the 53 7's were correct, see the table below.

Number	0	1	2	3	4	5	6	7	8	9
/Percent	%	%	%	%	%	%	%	%	%	%
100	8	8	12	11	10	8	9	8	12	14
	8.00	8.00	12.00	11.00	10.00	8.00	9.00	8.00	12.00	14.00
200	19	20	24	19	22	20	16	12	25	23
	9.50	10.00	12.00	9.50	11.00	10.00	8.00	6.00	12.50	11.50
300	26	30	35	31	37	27	31	19	34	30
	8.67	10.00	11.67	10.33	12.33	9.00	10.33	6.33	11.33	10.00
400	39	43	44	39	47	39	42	24	44	39
	9.75	10.75	11.00	9.75	11.75	9.75	10.50	6.00	11.00	9.75
500	45	59	54	50	53	50	48	36	53	52
	9.00	11.80	10.80	10.00	10.60	10.00	9.60	7.20	10.60	10.40
600	59	62	66	62	63	57	64	44	59	64
	9.83	10.33	11.00	10.33	10.50	9.50	10.67	7.33	9.83	10.67
700	72	78	74	72	71	62	70	52	71	78
	10.29	11.14	10.57	10.29	10.14	8.86	10.00	7.43	10.14	11.14
707	74	78	74	72	71	64	70	53	72	79
	10.47	11.03	10.47	10.18	10.04	9.05	9.90	7.50	10.18	11.17

The following table is the first correct 1,000 digits in a 100 step.

Number	0	1	2	3	4	5	6	7	8	9
/Percent	%	%	%	%	%	%	%	%	%	%
100	8	8	12	11	10	8	9	8	12	14
	8.00	8.00	12.00	11.00	10.00	8.00	9.00	8.00	12.00	14.00
200	19	20	24	19	22	20	16	12	25	23
	9.50	10.00	12.00	9.50	11.00	10.00	8.00	6.00	12.50	11.50
300	26	30	35	31	37	27	31	19	34	30
	8.67	10.00	11.67	10.33	12.33	9.00	10.33	6.33	11.33	10.00
400	39	43	44	39	47	39	42	24	44	39
	9.75	10.75	11.00	9.75	11.75	9.75	10.50	6.00	11.00	9.75
500	45	59	54	50	53	50	48	36	53	52
	9.00	11.80	10.80	10.00	10.60	10.00	9.60	7.20	10.60	10.40
600	54	67	64	63	64	55	62	50	59	62
	9.00	11.17	10.67	10.50	10.67	9.17	10.33	8.33	9.83	10.33
700	64	76	75	71	73	67	70	65	68	71
	9.14	10.86	10.71	10.14	10.43	9.57	10.00	9.29	9.71	10.14
800	74	92	83	79	80	73	77	75	76	91
	9.25	11.50	10.38	9.88	10.00	9.13	9.63	9.38	9.50	11.38
900	86	103	93	94	89	84	85	83	88	95
	9.56	11.44	10.33	10.44	9.89	9.33	9.44	9.22	9.78	10.56
1000	93	116	103	102	93	97	94	95	101	106
	9.30	11.60	10.30	10.20	9.30	9.70	9.40	9.50	10.10	10.60

The complete listing of the first 10,000,000 digits can be found at ["engert.us/erwin/miscellaneous/The distribution digits of pi.pdf"](http://engert.us/erwin/miscellaneous/The%20distribution%20digits%20of%20pi.pdf)
 The term listing for calculating PI is at ["engert.us/erwin/miscellaneous/PI.pdf"](http://engert.us/erwin/miscellaneous/PI.pdf)
 And continued fraction is at ["engert.us/erwin/miscellaneous/Continued fraction for pi.pdf"](http://engert.us/erwin/miscellaneous/Continued%20fraction%20for%20pi.pdf)

Here is how to calculate PI by yourself without a calculator or a computer. Any of the following formulas at the bottom of page 19 can be used to calculate the number PI. One of the most used formula is $PI = 4 * (4 * ATN(1/5) - ATN(1/239))$. The function $ATN(X) = X - X^3/3 + X^5/5 - X^7/7 + X^9/9$ etc. can be used to calculate the ATN function. The process continues until the value reaches enough 0's and no longer affects the total sum. See the next page how it works to produce the values needed. There are six more samples on pages 11, 12, 13, 14, 15, 16 & 17.

If you would like to see if you are able to do a small sample of what William Shanks did, but on a much larger scale. To do it correctly all the work must be done with paper and pencil only, no calculating type device, like a calculator or computer even an abacus. This would require you to know how to do long division without any aid other than your own mind plus paper and pencil. Can you do this challenge or will you just say it is too hard I am not capable to do the task, so which are you. Without looking at the following page except to see if you can do the first two terms correct. The work can be done in several days if you work long, hard and steady, or use a group effort to speed up the work but that means you did not do all the work yourself. There are some short cuts starting with doing the powers of 1/5, after the first term divide by 25 from sub term to sub term. In this case multiply by 4 and divide by 100, which is the same as shifting to the right by two digits. Then you still must do the division by the term number i.e. 3, 5, 7 etc.

These formulas are sometime referred to as Machin-like formula. There are many longer formulas that take more total terms to reach the same number of digits. Remember to alternate the addition and subtraction of every other term. Have fun with this problem. If you want you could use any of the following formulas listed on page 19 with further detailed information.

An early calculation not mentioned as often also had the same error as his 1873 work. The total term listing for 530 digits with 607 digits for PI is found in William Shanks 1853 calculation book "Contributions to mathematics, comprising chiefly the rectification of the circle". If someone would like to figure out where the error is in which term has the error. The next entry I have supplied term level for PI to 720 digits. One error was found to be in the term $(1/5)^{497/497}$ he omitted a 0 in the term in the 531 digit he had 8482897 instead of 80482897, his 1853 book just missed displaying this error. This 0 would repeat at digit 741. This error only corrects an extra 38 digits, $1/497$ has a repeat pattern of 210 and the $(1/5)^{497}$ fills more than the 707 digit position, the repeat starts at 708 digits. In the first series there still are more errors, and there is still an error in the second series. Due to William Shanks error all new records have to be calculated using two different methods, so his work was not in vain, even in 1947 they knew they needed to do two calculations see page 7. If you would like to see the detailed listing of each term and sub term William Shanks went through see the following web link "engert.us/erwin/miscellaneous/PI.pdf".

The following eight pages are listings of seven different methods to calculate 40 digits PI which are reduce to 35 correct digits. They require 35, 31, 34, 34, 47, 53 and 29 terms for the same number of digits of calculation; there is very little savings in work between the first four formulas while the next two are larger and the last is one digit shorter.

Here are the results of formula 8 on page 19. Here is a term by term breakdown for the solution of computing PI to 35 digits. Note how each term is getting closer to 0. You must always calculate more digits than required to for the accuracy you desired hence 40 digits to get a 35 digit answer. The reason I picked 35 digits was that Ludolph Van Ceulen (1540 - 1610) spent most of his life working out PI to 35 decimal places. PI is sometimes known as Ludolph's Constant the value is carved on his tombstone.

(1/5)=	+	0.2000000000	0000000000	0000000000	0000000000
(1/5) ^ 3/3=	-	0.0026666666	6666666666	6666666666	6666666666
(1/5) ^ 5/5=	+	0.0000640000	0000000000	0000000000	0000000000
(1/5) ^ 7/7=	-	0.0000018285	7142857142	8571428571	4285714285
(1/5) ^ 9/9=	+	0.0000000568	8888888888	8888888888	8888888888
(1/5) ^ 11/11=	-	0.0000000018	6181818181	8181818181	8181818181
(1/5) ^ 13/13=	+	0.0000000000	6301538461	5384615384	6153846153
(1/5) ^ 15/15=	-	0.0000000000	0218453333	3333333333	3333333333
(1/5) ^ 17/17=	+	0.0000000000	0007710117	6470588235	2941176470
(1/5) ^ 19/19=	-	0.0000000000	0000275941	0526315789	4736842105
(1/5) ^ 21/21=	+	0.0000000000	0000009986	4380952380	9523809523
(1/5) ^ 23/23=	-	0.0000000000	0000000364	7220869565	2173913043
(1/5) ^ 25/25=	+	0.0000000000	0000000013	4217728000	0000000000
(1/5) ^ 27/27=	-	0.0000000000	0000000000	4971026962	9629629629
(1/5) ^ 29/29=	+	0.0000000000	0000000000	0185127900	6896551724
(1/5) ^ 31/31=	-	0.0000000000	0000000000	0006927366	6064516129
(1/5) ^ 33/33=	+	0.0000000000	0000000000	0000260301	0482424242
(1/5) ^ 35/35=	-	0.0000000000	0000000000	0000009817	0681051428
(1/5) ^ 37/37=	+	0.0000000000	0000000000	0000000371	4566310054
(1/5) ^ 39/39=	-	0.0000000000	0000000000	0000000014	0963029202
(1/5) ^ 41/41=	+	0.0000000000	0000000000	0000000000	5363471355
(1/5) ^ 43/43=	-	0.0000000000	0000000000	0000000000	0204560302
(1/5) ^ 45/45=	+	0.0000000000	0000000000	0000000000	0007818749
(1/5) ^ 47/47=	-	0.0000000000	0000000000	0000000000	0000299441
(1/5) ^ 49/49=	+	0.0000000000	0000000000	0000000000	0000011488
(1/5) ^ 51/51=	-	0.0000000000	0000000000	0000000000	0000000441
(1/5) ^ 53/53=	+	0.0000000000	0000000000	0000000000	0000000016

ATN (1/5)=	+	0.1973955598	4988075837	0049765194	7902934477
4*ATN (1/5)=	+	0.7895822393	9952303348	0199060779	1611737908

(1/239)=	+	0.0041841004	1841004184	1004184100	4184100418
(1/239) ^ 3/3=	-	0.0000000244	1659178708	3803627411	8923012459
(1/239) ^ 5/5=	+	0.0000000000	0025647231	4424647365	7052071108
(1/239) ^ 7/7=	-	0.0000000000	0000000320	7130657784	6947170443
(1/239) ^ 9/9=	+	0.0000000000	0000000000	0043669315	2440391897
(1/239) ^ 11/11=	-	0.0000000000	0000000000	0000000625	5044509921
(1/239) ^ 13/13=	+	0.0000000000	0000000000	0000000000	0092658216
(1/239) ^ 15/15=	-	0.0000000000	0000000000	0000000000	0000001405

ATN (1/239)=	+	0.0041840760	0207472386	4538214959	2854527411
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4*ATN (1/5) -ATN (1/239) =PI/4=	+	0.7853981633	9744830961	5660845819	8757210497
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4* (4*ATN (1/5) -ATN (1/239)) =PI=	+	3.1415926535	8979323846	2643383279	5028841988
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The number PI correct to 40 positions after the decimal point is.

+ 3.1415926535 8979323846 2643383279 5028841971

The total number of number of terms is 35. The error of the computed value and the correct is 17 parts in the last digit, which effects the last two digits.

Here are the results of formula 12 on page 19. $PI=176*ATN(1/57)+28*ATN(1/239)-48*ATN(1/682)+96*ATN(1/12943)$

(1/57)=	+	0.0175438596	4912280701	7543859649	1228070175
(1/57) ^ 3/3=	-	0.0000017999	2404320537	6733101863	1013771218
(1/57) ^ 5/5=	+	0.0000000003	3239594519	0281945171	1658543632
(1/57) ^ 7/7=	-	0.0000000000	0007307653	8976890020	0437861879
(1/57) ^ 9/9=	+	0.0000000000	0000001749	3785193332	3121749086
(1/57) ^ 11/11=	-	0.0000000000	0000000000	4405385342	0630350478
(1/57) ^ 13/13=	+	0.0000000000	0000000000	0001147317	2517625159
(1/57) ^ 15/15=	-	0.0000000000	0000000000	0000000306	0454349628
(1/57) ^ 17/17=	+	0.0000000000	0000000000	0000000000	0831148321
(1/57) ^ 19/19=	-	0.0000000000	0000000000	0000000000	0000228888
(1/57) ^ 21/21=	+	0.0000000000	0000000000	0000000000	0000000063

ATN (1/57)=	+	0.0175420600	5740248778	1496767938	6820574345
44*ATN (1/57)=	+	0.7718506425	2570946238	5857789302	0105271180
(1/239)=	+	0.0041841004	1841004184	1004184100	4184100418
(1/239) ^ 3/3=	-	0.0000000244	1659178708	3803627411	8923012459
(1/239) ^ 5/5=	+	0.0000000000	0025647231	4424647365	7052071108
(1/239) ^ 7/7=	-	0.0000000000	0000000320	7130657784	6947170443
(1/239) ^ 9/9=	+	0.0000000000	0000000000	0043669315	2440391897
(1/239) ^ 11/11=	-	0.0000000000	0000000000	0000000625	5044509921
(1/239) ^ 13/13=	+	0.0000000000	0000000000	0000000000	0092658216
(1/239) ^ 15/15=	-	0.0000000000	0000000000	0000000000	0000001405

ATN (1/239)=	+	0.0041840760	0207472386	4538214959	2854527411
7*ATN (1/239)=	+	0.0292885320	1452306705	1767504714	9981691877

44*ATN (1/57) +7*ATN (1/239) =	+	0.8011391745	4023252943	7625294017	0086963057
(1/682)=	+	0.0014662756	5982404692	0821114369	5014662756
(1/682) ^ 3/3=	-	0.0000000010	5081344603	7362739700	3196061705
(1/682) ^ 5/5=	+	0.0000000000	0000135552	6843642593	4671618574
(1/682) ^ 7/7=	-	0.0000000000	0000000000	2081667382	7725716570
(1/682) ^ 9/9=	+	0.0000000000	0000000000	0000003480	9526729330
(1/682) ^ 11/11=	-	0.0000000000	0000000000	0000000000	0061232105
(1/682) ^ 13/13=	+	0.0000000000	0000000000	0000000000	0000000111

ATN (1/682)=	+	0.0014662746	0901195640	8220353360	8230000391
12*ATN (1/682)=	+	0.0175952953	0814347689	8644240329	8760004692

44*ATN (1/57) +7*ATN (1/239) -12*ATN (1/682)	+	0.7835438792	3208905253	8981053687	1326958365
(1/12943)=	+	0.0000772618	4037703778	1039944371	4749285327
(1/12943) ^ 3/3=	-	0.0000000000	0015373540	3395577160	5625671432
(1/12943) ^ 5/5=	+	0.0000000000	0000000000	0550624130	2997107927
(1/12943) ^ 7/7=	-	0.0000000000	0000000000	0000000002	3477794761
(1/12943) ^ 9/9=	+	0.0000000000	0000000000	0000000000	0000000109

ATN (1/12943)=	+	0.0000772618	4022330237	8194991338	8642927170
24*ATN (1/12943)=	+	0.0018542841	6535925707	6679792132	7430252080

44*ATN (1/57) +7*ATN (1/239) -12*ATN (1/682) +24*ATN (1/12943) =PI/4=	+	0.7853981633	9744830961	5660845819	8757210445

4* (44*ATN (1/57) +7*ATN (1/239) -12*ATN (1/682) +24*ATN (1/12943)) =PI=	+	3.1415926535	8979323846	2643383279	5028841780

THE NUMBER PI TO 35 POSITIONS AFTER THE DECIMAL POINT.

+ 3.1415926535 8979323846 2643383279 50288

The total number of number of terms is 31 a savings of 4 terms over page 10. The error of the computed value and the correct is 191 parts in the last digit, which effects the last three digits.

Here are the results of formula 9 on page 19; the first series is easy to do. $PI=32 * ATN(1/10) - 4 * ATN(1/239) - 16 * ATN(1/515)$

(1/10)=	+	0.1000000000	0000000000	0000000000	0000000000
(1/10) ^ 3/3=	-	0.0003333333	3333333333	3333333333	3333333333
(1/10) ^ 5/5=	+	0.0000020000	0000000000	0000000000	0000000000
(1/10) ^ 7/7=	-	0.000000142	8571428571	4285714285	7142857142
(1/10) ^ 9/9=	+	0.0000000001	1111111111	1111111111	1111111111
(1/10) ^ 11/11=	-	0.0000000000	0090909090	9090909090	9090909090
(1/10) ^ 13/13=	+	0.0000000000	0000769230	7692307692	3076923076
(1/10) ^ 15/15=	-	0.0000000000	0000066666	6666666666	6666666666
(1/10) ^ 17/17=	+	0.0000000000	0000000058	8235294117	6470588235
(1/10) ^ 19/19=	-	0.0000000000	0000000000	5263157894	7368421052
(1/10) ^ 21/21=	+	0.0000000000	0000000000	0047619047	6190476190
(1/10) ^ 23/23=	-	0.0000000000	0000000000	0000434782	6086956521
(1/10) ^ 25/25=	+	0.0000000000	0000000000	0000004000	0000000000
(1/10) ^ 27/27=	-	0.0000000000	0000000000	0000000037	0370370370
(1/10) ^ 29/29=	+	0.0000000000	0000000000	0000000000	3448275862
(1/10) ^ 31/31=	-	0.0000000000	0000000000	0000000000	0032258064
(1/10) ^ 33/33=	+	0.0000000000	0000000000	0000000000	0000303030
(1/10) ^ 35/35=	-	0.0000000000	0000000000	0000000000	0000002857
(1/10) ^ 37/37=	+	0.0000000000	0000000000	0000000000	0000000027

ATN(1/10)=	+	0.0996686524	9116202737	8446119878	0205902436

8*ATN(1/10)=	+	0.7973492199	2929621902	7568959024	1647219488
(1/239)=	+	0.0041841004	1841004184	1004184100	4184100418
(1/239) ^ 3/3=	-	0.0000000244	1659178708	3803627411	8923012459
(1/239) ^ 5/5=	+	0.0000000000	0025647231	4424647365	7052071108
(1/239) ^ 7/7=	-	0.0000000000	0000000320	7130657784	6947170443
(1/239) ^ 9/9=	+	0.0000000000	0000000000	0043669315	2440391897
(1/239) ^ 11/11=	-	0.0000000000	0000000000	0000000625	5044509921
(1/239) ^ 13/13=	+	0.0000000000	0000000000	0000000000	0092658216
(1/239) ^ 15/15=	-	0.0000000000	0000000000	0000000000	0000001405

ATN(1/239)=	+	0.0041840760	0207472386	4538214959	2854527411

8*ATN(1/10) - ATN(1/239) =	+	0.7931651439	2722149516	3030744064	8792692077
(1/515)=	+	0.0019417475	7281553398	0582524271	8446601941
(1/515) ^ 3/3=	-	0.0000000024	4037775827	5092192895	9993362172
(1/515) ^ 5/5=	+	0.0000000000	0000552069	6220058649	5079181811
(1/515) ^ 7/7=	-	0.0000000000	0000000001	4867959064	5641827233
(1/515) ^ 9/9=	+	0.0000000000	0000000000	0000043600	5963288818
(1/515) ^ 11/11=	-	0.0000000000	0000000000	0000000000	1345017067
(1/515) ^ 13/13=	+	0.0000000000	0000000000	0000000000	0000004291

ATN(1/515)=	+	0.0019417451	3244329638	6842474561	2508870389

4*ATN(1/515)=	+	0.0077669805	2977318554	7369898245	0035481556

8*ATN(1/10) - ATN(1/239) - 4*ATN(1/515) = PI/4 =	+	0.7853981633	9744830961	5660845819	8757210521

4* (8*ATN(1/10) - ATN(1/239) - 4*ATN(1/515)) = PI =	+	3.1415926535	8979323846	2643383279	5028842084
THE NUMBER PI TO 35 POSITIONS AFTER THE DECIMAL POINT.	+	3.1415926535	8979323846	2643383279	50288

The total number of number of terms is 34, a savings of 1 term over page 10. The error of the computed value and the correct is 113 parts in the last digit, which effects the last four digits.

Here are the results of formula 10 on page 19 and is a fast formula too. $PI=48*ATN(1/18)+32*ATN(1/57)-20*ATN(1/239)$

(1/18)=	+	0.0555555555	5555555555	5555555555	5555555555
(1/18)^ 3/3=	-	0.0000571559	2135345221	7649748513	9460448102
(1/18)^ 5/5=	+	0.0000001058	4429880268	9291943978	7295297126
(1/18)^ 7/7=	-	0.0000000002	3334281041	1572513104	0095448185
(1/18)^ 9/9=	+	0.0000000000	0056015077	9451648694	0082533654
(1/18)^ 11/11=	-	0.0000000000	0000141452	2170332446	1969905388
(1/18)^ 13/13=	+	0.0000000000	0000000369	4146218816	9297642203
(1/18)^ 15/15=	-	0.0000000000	0000000000	9881461079	1399355833
(1/18)^ 17/17=	+	0.0000000000	0000000000	0026910297	0564813060
(1/18)^ 19/19=	-	0.0000000000	0000000000	0000074313	6858284896
(1/18)^ 21/21=	+	0.0000000000	0000000000	0000000207	5191109261
(1/18)^ 23/23=	-	0.0000000000	0000000000	0000000000	5847962063
(1/18)^ 25/25=	+	0.0000000000	0000000000	0000000000	0016605324
(1/18)^ 27/27=	-	0.0000000000	0000000000	0000000000	0000047454
(1/18)^ 29/29=	+	0.0000000000	0000000000	0000000000	0000000136

ATN(1/18)= + 0.0554985052 4571683555 7198148092 2372104398

12*ATN(1/18)=	+	0.6659820629	4860202668	6377777106	8465252776
(1/57)=	+	0.0175438596	4912280701	7543859649	1228070175
(1/57)^ 3/3=	-	0.0000017999	2404320537	6733101863	1013771218
(1/57)^ 5/5=	+	0.0000000003	3239594519	0281945171	1658543632
(1/57)^ 7/7=	-	0.0000000000	0007307653	8976890020	0437861879
(1/57)^ 9/9=	+	0.0000000000	0000001749	3785193332	3121749086
(1/57)^ 11/11=	-	0.0000000000	0000000000	4405385342	0630350478
(1/57)^ 13/13=	+	0.0000000000	0000000000	0001147317	2517625159
(1/57)^ 15/15=	-	0.0000000000	0000000000	0000000306	0454349628
(1/57)^ 17/17=	+	0.0000000000	0000000000	0000000000	0831148321
(1/57)^ 19/19=	-	0.0000000000	0000000000	0000000000	0000228888
(1/57)^ 21/21=	+	0.0000000000	0000000000	0000000000	0000000063

ATN(1/57)= + 0.0175420600 5740248778 1496767938 6820574345

8*ATN(1/57)= + 0.1403364804 5921990225 1974143509 4564594760

12*ATN(1/18)+8*ATN(1/57)	+	0.8063185434	0782192893	8351920616	3029847536
(1/239)=	+	0.0041841004	1841004184	1004184100	4184100418
(1/239)^ 3/3=	-	0.0000000244	1659178708	3803627411	8923012459
(1/239)^ 5/5=	+	0.0000000000	0025647231	4424647365	7052071108
(1/239)^ 7/7=	-	0.0000000000	0000000320	7130657784	6947170443
(1/239)^ 9/9=	+	0.0000000000	0000000000	0043669315	2440391897
(1/239)^ 11/11=	-	0.0000000000	0000000000	0000000625	5044509921
(1/239)^ 13/13=	+	0.0000000000	0000000000	0000000000	0092658216
(1/239)^ 15/15=	-	0.0000000000	0000000000	0000000000	0000001405

ATN(1/239)= + 0.0041840760 0207472386 4538214959 2854527411

5*ATN(1/1985) + 0.0209203800 1037361932 2691074796 4272637055

12*ATN(1/18)+8*ATN(1/57)-5*ATN(1/239)=
 + 0.7853981633 9744830961 5660845819 8757210481

4*(12*ATN(1/18)+8*ATN(1/57)-5*ATN(1/239))=PI=
 + 3.1415926535 8979323846 2643383279 5028841924

THE NUMBER PI TO 35 POSITIONS AFTER THE DECIMAL POINT.

+ 3.1415926535 8979323846 2643383279 50288

The total number of number of terms is 34, a savings of 1 term over page 10. The error of the computed value and the correct is 47 parts in the last digit, which effects the last two digits.

Here are the results of formula 4 on page 19. William Rutherford used $\pi/4=4*ATN(1/5)-ATN(1/70)+ATN(1/99)$ To calculate 208 digits while only 152 were correct solution.

```
(1/5)= + 0.2000000000 0000000000 0000000000 0000000000
(1/5) ^ 3/3= - 0.0026666666 6666666666 6666666666 6666666666
(1/5) ^ 5/5= + 0.0000640000 0000000000 0000000000 0000000000
(1/5) ^ 7/7= - 0.0000018285 7142857142 8571428571 4285714285
(1/5) ^ 9/9= + 0.0000000568 8888888888 8888888888 8888888888
(1/5) ^ 11/11= - 0.0000000018 6181818181 8181818181 8181818181
(1/5) ^ 13/13= + 0.0000000000 6301538461 5384615384 6153846153
(1/5) ^ 15/15= - 0.0000000000 0218453333 3333333333 3333333333
(1/5) ^ 17/17= + 0.0000000000 0007710117 6470588235 2941176470
(1/5) ^ 19/19= - 0.0000000000 0000275941 0526315789 4736842105
(1/5) ^ 21/21= + 0.0000000000 0000009986 4380952380 9523809523
(1/5) ^ 23/23= - 0.0000000000 0000000364 7220869565 2173913043
(1/5) ^ 25/25= + 0.0000000000 0000000013 4217728000 0000000000
(1/5) ^ 27/27= - 0.0000000000 0000000000 4971026962 9629629629
(1/5) ^ 29/29= + 0.0000000000 0000000000 0185127900 6896551724
(1/5) ^ 31/31= - 0.0000000000 0000000000 0006927366 6064516129
(1/5) ^ 33/33= + 0.0000000000 0000000000 0000260301 0482424242
(1/5) ^ 35/35= - 0.0000000000 0000000000 0000009817 0681051428
(1/5) ^ 37/37= + 0.0000000000 0000000000 0000000371 4566310054
(1/5) ^ 39/39= - 0.0000000000 0000000000 0000000014 0963029202
(1/5) ^ 41/41= + 0.0000000000 0000000000 0000000000 5363471355
(1/5) ^ 43/43= - 0.0000000000 0000000000 0000000000 0204560302
(1/5) ^ 45/45= + 0.0000000000 0000000000 0000000000 0007818749
(1/5) ^ 47/47= - 0.0000000000 0000000000 0000000000 0000299441
(1/5) ^ 49/49= + 0.0000000000 0000000000 0000000000 0000011488
(1/5) ^ 51/51= - 0.0000000000 0000000000 0000000000 0000000441
(1/5) ^ 53/53= + 0.0000000000 0000000000 0000000000 0000000016
ATN(1/5)= + 0.1973955598 4988075837 0049765194 7902934477
```

```
-----
4*ATN(1/5)= + 0.7895822393 9952303348 0199060779 1611737908
(1/70)= + 0.0142857142 8571428571 4285714285 7142857142
(1/70) ^ 3/3= - 0.0000009718 1729834791 0592808551 9922254616
(1/70) ^ 5/5= + 0.0000000001 1899803653 2397215445 9451419051
(1/70) ^ 7/7= - 0.0000000000 0001734665 2555743034 3215663472
(1/70) ^ 9/9= + 0.0000000000 0000000275 3436913610 0054478676
(1/70) ^ 11/11= - 0.0000000000 0000000000 0459757555 1482383864
(1/70) ^ 13/13= + 0.0000000000 0000000000 0000079392 9844055042
(1/70) ^ 15/15= - 0.0000000000 0000000000 0000000014 0422965615
(1/70) ^ 17/17= + 0.0000000000 0000000000 0000000000 0025286248
(1/70) ^ 19/19= - 0.0000000000 0000000000 0000000000 0000004617
```

```
-----
ATN(1/70)= + 0.0142847425 8739663043 6511613579 1474823975
4*ATN(1/5)-ATN(1/70)= + 0.7752974968 1212640304 3687447200 0136913933
(1/99)= + 0.0101010101 0101010101 0101010101 0101010101
(1/99) ^ 3/3= - 0.0000003435 3671737612 1518555964 0207125157
(1/99) ^ 5/5= + 0.0000000000 2103071425 6267004502 9668821985
(1/99) ^ 7/7= - 0.0000000000 0000153269 4495916379 1233377703
(1/99) ^ 9/9= + 0.0000000000 0000000012 1630009085 4068616962
(1/99) ^ 11/11= - 0.0000000000 0000000000 0010153602 8955177278
(1/99) ^ 13/13= + 0.0000000000 0000000000 0000000876 5952599082
(1/99) ^ 15/15= - 0.0000000000 0000000000 0000000000 0775141201
(1/99) ^ 17/17= + 0.0000000000 0000000000 0000000000 0000069783
(1/99) ^ 19/19= - 0.0000000000 0000000000 0000000000 0000000006
```

```
-----
ATN(1/99)= + 0.0101006665 8532190657 1973398619 8620296568
```

```
-----
4*ATN(1/5)-ATN(1/70)+ATN(1/99)=PI/4= + 0.7853981633 9744830961 5660845819 8757210501
```

```
-----
4*(4*ATN(1/5)-ATN(1/70)+ATN(1/99))=PI= + 3.1415926535 8979323846 2643383279 5028842004
```

THE NUMBER PI TO 35 POSITIONS AFTER THE DECIMAL POINT.

```
+ 3.1415926535 8979323846 2643383279 50288
```

The total number of number of terms is 47; this is larger by 12 terms over page 10. The error of the computed value and the correct is 33 parts in the last digit, which effects the last four digits.

Here are the results of formula 3 on page 19; this was used by D F Ferguson in 1945

```

PI=12*ATN(1/4)+4*ATN(1/20)+4*ATN(1/1985)
(1/4)= + 0.2500000000 0000000000 0000000000 0000000000
(1/4)^ 3/3= - 0.0052083333 3333333333 3333333333 3333333333
(1/4)^ 5/5= + 0.00001953125 0000000000 0000000000 0000000000
(1/4)^ 7/7= - 0.0000087193 0803571428 5714285714 2857142857
(1/4)^ 9/9= + 0.0000004238 5525173611 1111111111 1111111111
(1/4)^ 11/11= - 0.0000000216 7441628196 0227272727 2727272727
(1/4)^ 13/13= + 0.0000000011 4624316875 7512019230 7692307692
(1/4)^ 15/15= - 0.0000000000 6208817164 1031901041 6666666666
(1/4)^ 17/17= + 0.0000000000 0342398005 3733376895 6801470588
(1/4)^ 19/19= - 0.0000000000 0019147256 8794300681 6663240131
(1/4)^ 21/21= + 0.0000000000 0001082731 7878249145 6894647507
(1/4)^ 23/23= - 0.0000000000 0000061786 3248487043 6398879341
(1/4)^ 25/25= + 0.0000000000 0000003552 7136788005 0092935562
(1/4)^ 27/27= - 0.0000000000 0000000205 5968564120 6602600437
(1/4)^ 29/29= + 0.0000000000 0000000011 9636101791 5039375456
(1/4)^ 31/31= - 0.0000000000 0000000000 6994852725 7129318323
(1/4)^ 33/33= + 0.0000000000 0000000000 0410682641 0929941037
(1/4)^ 35/35= - 0.0000000000 0000000000 0024200941 3501228668
(1/4)^ 37/37= + 0.0000000000 0000000000 0001430798 8973890208
(1/4)^ 39/39= - 0.0000000000 0000000000 0000084839 0371849259
(1/4)^ 41/41= + 0.0000000000 0000000000 0000005043 7842228843
(1/4)^ 43/43= - 0.0000000000 0000000000 0000000300 5743504916
(1/4)^ 45/45= + 0.0000000000 0000000000 0000000017 9509681543
(1/4)^ 47/47= - 0.0000000000 0000000000 0000000001 0741935730
(1/4)^ 49/49= + 0.0000000000 0000000000 0000000000 0643968085
(1/4)^ 51/51= - 0.0000000000 0000000000 0000000000 0038669652
(1/4)^ 53/53= + 0.0000000000 0000000000 0000000000 0002325651
(1/4)^ 55/55= - 0.0000000000 0000000000 0000000000 0000140067
(1/4)^ 57/57= + 0.0000000000 0000000000 0000000000 0000008447
(1/4)^ 59/59= - 0.0000000000 0000000000 0000000000 0000000510
(1/4)^ 61/61= + 0.0000000000 0000000000 0000000000 0000000030
(1/4)^ 63/63= - 0.0000000000 0000000000 0000000000 0000000001
ATN(1/4)= + 0.2449786631 2686415417 2082481211 2758109142
-----
3*ATN(1/4)= + 0.7349359893 8059246251 6247443633 8274327426
(1/20)= + 0.0500000000 0000000000 0000000000 0000000000
(1/20)^ 3/3= - 0.0000416666 6666666666 6666666666 6666666666
(1/20)^ 5/5= + 0.0000000625 0000000000 0000000000 0000000000
(1/20)^ 7/7= - 0.0000000001 1160714285 7142857142 8571428571
(1/20)^ 9/9= + 0.0000000000 0021701388 8888888888 8888888888
(1/20)^ 11/11= - 0.0000000000 0000044389 2045454545 4545454545
(1/20)^ 13/13= + 0.0000000000 0000000093 9002403846 1538461538
(1/20)^ 15/15= - 0.0000000000 0000000000 2034505208 3333333333
(1/20)^ 17/17= + 0.0000000000 0000000000 0004487879 1360294117
(1/20)^ 19/19= - 0.0000000000 0000000000 0000010038 6770148026
(1/20)^ 21/21= + 0.0000000000 0000000000 0000000022 7065313430
(1/20)^ 23/23= - 0.0000000000 0000000000 0000000000 0518301258
(1/20)^ 25/25= + 0.0000000000 0000000000 0000000000 0001192092
(1/20)^ 27/27= - 0.0000000000 0000000000 0000000000 0000002759
(1/20)^ 29/29= + 0.0000000000 0000000000 0000000000 0000000006
ATN(1/20)= + 0.0499583957 2194276141 0006287034 8448814913
-----
3*ATN(1/4)+ATN(1/20)= + 0.7848943851 0253522392 6253730668 6723142339
(1/1985)= + 0.0005037783 3753148614 6095717884 1309823677
(1/1985)^ 3/3= - 0.0000000000 4261840694 6429552127 5830596925
(1/1985)^ 5/5= + 0.0000000000 0000000648 9742125857 7191090736
(1/1985)^ 7/7= - 0.0000000000 0000000000 0001176463 2958519807
(1/1985)^ 9/9= + 0.0000000000 0000000000 0000000000 2322270956
(1/1985)^ 11/11= - 0.0000000000 0000000000 0000000000 0000000482
ATN(1/1985)= + 0.0005037782 9491308568 9407115151 2034068155
-----
3*ATN(1/4)+ATN(1/20)+ATN(1/1985)=PI/4= + 0.7853981633 9744830961 5660845819 8757210494
4*(3*ATN(1/4)+ATN(1/20)+ATN(1/1985))=PI= + 3.1415926535 8979323846 2643383279 5028841976
THE NUMBER PI TO 35 POSITIONS AFTER THE DECIMAL POINT.
+ 3.1415926535 8979323846 2643383279 50288

```

The total number of number of terms is 53; this is larger by 18 terms over page 10, this is not a savings. The error of the computed value and the correct is 5 parts in the last digit, which effects the last digit. This is a much slower converging series, Ferguson used the fact that 1/4 and 1/20 are somewhat related. For the calculations of 820 digits the error in the last digit is 62 units which effects the last 4 digits, they over did it by dropping the last 10 digits and even questioned the next two leaving 808 digits.

Here are the results of formula 13 on page 19; this is a fast formula all though it has 6 series.

$$PI=732*ATN(1/239)+128*ATN(1/1023)-272*ATN(1/5832)+48*ATN(1/110443)-48*ATN(1/4841182)-400*ATN(1/6826318))$$

(1/239) =	+	0.0041841004	1841004184	1004184100	4184100418
(1/239)^ 3/3 =	-	0.0000000244	1659178708	3803627411	8923012459
(1/239)^ 5/5 =	+	0.0000000000	0025647231	4424647365	7052071108
(1/239)^ 7/7 =	-	0.0000000000	0000000320	7130657784	6947170443
(1/239)^ 9/9 =	+	0.0000000000	0000000000	0043669315	2440391897
(1/239)^ 11/11 =	-	0.0000000000	0000000000	0000000625	5044509921
(1/239)^ 13/13 =	+	0.0000000000	0000000000	0000000000	0092658216
(1/239)^ 15/15 =	-	0.0000000000	0000000000	0000000000	0000001405

ATN(1/239) =	+	0.0041840760	0207472386	4538214959	2854527411

183*ATN(1/239) =	+	0.7656859083	7967446721	0493337549	2378516213
(1/1023) =	+	0.0009775171	0654936461	3880742913	0009775171
(1/1023)^ 3/3 =	-	0.0000000003	1135213215	9218589540	8354388653
(1/1023)^ 5/5 =	+	0.0000000000	0000017850	5592578448	5224246067
(1/1023)^ 7/7 =	-	0.0000000000	0000000000	0121835128	0269269191
(1/1023)^ 9/9 =	+	0.0000000000	0000000000	0000000090	5475673699
(1/1023)^ 11/11 =	-	0.0000000000	0000000000	0000000000	0000707905

ATN(1/1023) =	+	0.0009775167	9519741096	0132896783	2085329188

32*ATN(1/1023) =	+	0.0312805374	4631715072	4252697062	6730534016

183*ATN(1/239)+32*ATN(1/1023) =	+	0.7969664458	2599161793	4746034611	9109050229
(1/5832) =	+	0.0001714677	6406035665	2949245541	8381344307
(1/5832)^ 3/3 =	-	0.0000000000	0168045233	8354946082	0247600964
(1/5832)^ 5/5 =	+	0.0000000000	0000000002	9644383237	4198067499
(1/5832)^ 7/7 =	-	0.0000000000	0000000000	0000000622	5573327783
(1/5832)^ 9/9 =	+	0.0000000000	0000000000	0000000000	0000142363

ATN(1/5832) =	+	0.0001714677	6237990434	4238682074	6758625422

68*ATN(1/5832) =	+	0.0116598078	4183349540	8230381077	9586528696

183*ATN(1/239)+32*ATN(1/1023) -68*ATN(1/5832)	+	0.7853066379	8415812252	6515653533	9522521533
(1/110443) =	+	0.0000090544	4437402098	8202058980	6506523727
(1/110443)^ 3/3 =	-	0.0000000000	0000024743	6725798950	0613954922
(1/110443)^ 5/5 =	+	0.0000000000	0000000000	0000012171	3575500655
(1/110443)^ 7/7 =	-	0.0000000000	0000000000	0000000000	0000007127

ATN(1/110443) =	+	0.0000090544	4437377355	1476272201	9468062333

12*ATN(1/110443) =	+	0.0001086533	3248528261	7715266423	3616747996

183*ATN(1/239)+32*ATN(1/1023) -68*ATN(1/5832)+12*ATN(1/110443)	+	0.7854152913	1664340514	4230919957	3139269529
(1/4841182) =	+	0.0000002065	6112494841	1359044134	2630787274
(1/4841182)^ 3/3 =	-	0.0000000000	0000000000	2937815485	2784030550
(1/4841182)^ 5/5 =	+	0.0000000000	0000000000	0000000000	0000752095

ATN(1/4841182) =	+	0.0000002065	6112494840	8421228648	9847508819

12*ATN(1/4841182) =	+	0.0000024787	3349938090	1054743787	8170105828

183*ATN(1/239)+32*ATN(1/1023) -68*ATN(1/5832)+12*ATN(1/110443) -12*ATN(1/4841182) =					


```

(1/6826318) = + 0.7854128125 8314402424 3176176169 4969163701
(1/6826318)^ 3/3 = + 0.0000001464 9185695714 7323051753 5221769627
(1/6826318)^ 5/5 = - 0.0000000000 0000000000 1047898450 0259785023
----- + 0.0000000000 0000000000 0000000000 0000134926
ATN(1/6826318) = + 0.0000001464 9185695714 6275153303 4962119530
-----
100*ATN(1/6826318) = + 0.0000146491 8569571462 7515330349 6211953000
-----
183*ATN(1/239)+32*ATN(1/1023)-68*ATN(1/5832)+12*ATN(1/110443)-12*ATN(1/4841182)-
100*ATN(1/6826318) = PI/4 = + 0.7853981633 9744830961 5660845819 8757210701
-----
4*(183*ATN(1/239)+32*ATN(1/1023)-68*ATN(1/5832)+12*ATN(1/110443)-12*ATN(1/4841182)-
100*ATN(1/6826318))= PI = + 3.1415926535 8979323846 2643383279 5028842804
THE NUMBER PI TO 35 POSITIONS AFTER THE DECIMAL POINT.
+ 3.1415926535 8979323846 2643383279 50288

```

The total number of number of terms is 29, a savings of 6 terms over page 10. The error of the computed value and the correct is 833 parts in the last digit, which effects the last four digits. To expand this to 100 digits the error increases 4265 parts in the last digit, which effects the last four digits. To expand this to 1000 digits the error increases 2889 parts in the last digit, which effects the last five digits. To the person who looks at the six fewer terms as a savings, the larger values are harder to do hand calculation and any gain is lost with the multiplying. The method used here is great for a computer doing 1 million digits not 40 digits by hand. Due to the large number of series with all the sub calculations created extra lines has resulted in requiring two pages to display the results.

The following three numbers came from William Shanks 1853 calculation book "Contributions to mathematics, comprising chiefly the rectification of the circle". If someone would like to figure out where the error is in which term has the error. It is known he did correct some errors before his 707 publication. Two years latter Richter calculated 500 digits of pi which William Shanks used to check his work, this was a little after the fact. Richter work is very seldom mentions most likely since 607 digits of 609 digits had been published.

ATN(1/5)=

0.1973955 5984988 0758370 0497651 9479029 3447585 1037878
 5210151 7688940 2410339 6997824 3785732 6978280 3728804
 4112628 1180736 9136010 4456479 8867942 3935574 7565495
 2163032 7005221 0747001 5645015 5600612 8618552 6633257
 3186928 0664389 6806189 5284058 2593112 4251613 2973139
 9339711 3233537 8217960 8417664 8310525 4730396 6572565
 0488878 1553093 8429057 9311695 9341928 5180636 4919697
 5194017 0856094 9527368 6737385 0840081 2367856 1580093
 2982251 4023246 6755492 1102670 4574378 8154748 3907997
 8985020 0752236 9683796 1392278 3541932 5572232 8413846
 4774413 5290970 5465122 4383026 9756051 8377**617 7816424**
2337830 3370181 9264880 2827768 6119150 9856067 5901213
5985563 6303434 7839296 last 8 digits different from 709 value

ATN(1/239)=

0.0041840 7600207 4723864 5382149 5928545 2741048 0653076
 3195082 7019612 8871817 7834142 2893273 7826058 1362290
 9454975 4506664 4486375 6052458 3947893 1186505 8922128
 8330928 0084627 1962330 7733759 4763460 3318473 4145703
 3198601 5454814 8059924 4983021 1460391 2539495 2760779
 6881558 8812733 9785334 6518045 7425481 3586746 4475197
 9102328 3097700 2064652 8276346 5329691 0481838 6543560
 7891959 1451232 2209446 3686276 6155208 3167964 2646574
 6551103 2510343 5262824 4512693 5567049 9684445 2479043
 3177283 9307086 3140193 8695195 0370586 4107708 5585540
 4522355 3881423 7677083 6515691 8252702 0022930 8954495
 0043585 4409344 9644014 2418724 9509228 3862395 4553335
 651**6494 2122006 8523882** all digits matched from the 709 value.

THE NUMBER PI TO 609 POSITION AFTER THE DECIMAL POINT=

3.1415926 5358979 3238462 6433832 7950288 4197169 3993751
 0582097 4944592 3078164 0628620 8998628 0348253 4211706
 7982148 0865132 8230664 7093844 6095505 8223172 5359408
 1284811 1745028 4102701 9385211 0555964 4622948 9549303
 8196442 8810975 6659334 4612847 5648233 7867831 6527120
 1909145 6485669 2346034 8610454 3266482 1339360 7260249
 1412737 2458700 6606315 5881748 8152092 0962829 2540917
 1536436 7892590 3600113 3053054 8820466 5213841 4695194
 1511609 4330572 7036575 9591953 0921861 1738193 2611793
 1051185 4807446 2379962 7495673 5188575 2724891 2279381
 8301194 9129833 6733624 4065664 3086021 39**50160 9244807**
7230943 6285530 9662027 5569397 9869502 2247499 6206074
9703041 2366929 1333208 Last 9 digits different from the 709 value with add 2 digits

For those who would like to know how many total terms it takes to calculate say 1000 digits here are the numbers for each series and the total. In the first sample, the first series takes 1655 terms while the second series takes 713 and the third series takes 552 for a total of 2920 total terms, which would mean it takes 2.9 terms to produce one digit. This formula is the one used by Zacharias Dasein 1844; this is the slowest converging solution I have found other than $4*ATN(1)$, it requiring five hundred terms to produce two digit results (3.14).

In 1949 ENIAC the first programmable computer and in 1972 I also used the formula $PI=4*(4*ATN(1/5)-ATN(1/239))$ on an IBM 1620 type 1, and programmed the instructions in machine language to produce 100 digits of PI just for the fun of it. I used punch cards to enter the instructions for the computer as I had no compiler and then loaded them into the computer. This computer had only 20,000 locations of memory, and each could store one decimal digit and no operating system; you loaded one program at a time. For those who know what punched cards and machine language are, the better you will understand what I am talking about? This computer could not add, subtract, multiply or divide, you had to load the addition and multiply tables first. The IBM 1620 was a very unique computer as other computers of the time, in fact it was a decimal computer for instructions, memory and calculations, so the answer needed no conversion from binary to decimal.

- 1 $PI=4*ATN(1/2)+4*ATN(1/5)+4*ATN(1/8)$
1655+713+552=2920
- 2 $PI=4*ATN(1/2)+4*ATN(1/3)$
1655+1044=2699
- 3 $PI=12*ATN(1/4)+4*ATN(1/20)+4*ATN(1/1985)$
828+383+151=1362
- 4 $PI=16*ATN(1/5)-4*ATN(1/70)+4*ATN(1/99)$
713+270+250=1233
- 5 $PI=24*ATN(1/8)+8*ATN(1/57)+4*ATN(1/239)$
552+284+210=1046
- 6 $PI=48*ATN(1/38)+80*ATN(1/57)+28*ATN(1/239)+96*ATN(1/268)$
316+284+210+205=1015
- 7 $PI=20*ATN(1/7)+8*ATN(3/79)$
590+351=941
- 8 $PI=16*ATN(1/5)-4*ATN(1/239)$
713+210=923
- 9 $PI=32*ATN(1/10)-4*ATN(1/239)-16*ATN(1/515)$
499+210+184=893
- 10 $PI=48*ATN(1/18)+32*ATN(1/57)-20*ATN(1/239)$
397+284+210=891
- 11 $PI=48*ATN(1/49)+128*ATN(1/57)-20*ATN(1/239)+48*ATN(1/110443)$
295+284+210+99=888
- 12 $PI=176*ATN(1/57)+28*ATN(1/239)-48*ATN(1/682)+96*ATN(1/12943)$
284+210+176+121=791
- 13 $PI=732*ATN(1/239)+128*ATN(1/1023)-272*ATN(1/5832)+48*ATN(1/110443)-$
 $48*ATN(1/4841182)-400*ATN(1/6826318)$
210+166+132+99+75+73=755

A new error within the 180 digits error

This is the investigation of the new error found in the 180 error digits; see the note on the bottom page 3. Starting with a block of ten digits from the two sub terms used to produce the final answer published by William Shanks. When I did this check it was for checking the two sub terms for possible typo errors and not the final answer, as it turned out. The method I have used in the first sample, to convert the two sub terms into the final answer is to multiply the first term by four then subtract the second term and finally to do another multiply by four. The error in the first two steps would produce error of 16 or 4 parts in a single digit, not the 11 parts as observed. In the third step there was a series of 6 3's in a row a miss carry on the first two would have created the desired which effects, this is most likely what happened.

In the second sample I first multiply the first series by 16 which would require an extra addition which produces a forth step then multiple the second series by four and subtract the two values. As you can see below both method gives the same answer, which they should but the second sample takes an extra step.

The first sample most likely

```
2 22133 3312 Carry
 56649 97826
      4 Times
-----
2 26599 91304

1 11001 1011 Borrow
2 26599 91304
 38821 94006 Subtraction
-----
1 87777 97298

3 33333 2133 Carry
1 87777 97298
      4 Times
-----
7 51111 89192
```

The second sample very unlikely

```
3 33255 4413 Carry from time 6 only the times one is just a simple shift
 56649 97826
      16 Times
-----
1 11111 1110 (Carry from addition only)
3 39899 86956
5 66499 78260 Addition the extra step
-----
9 06399 65216

1 33003 1002 Carry
 38821 94006
      4 Times
-----
1 55287 76024

1 0001 1010 Borrow
9 06399 65216
1 55287 76024 Subtract
-----
7 51111 89192
```

Here are the results of from formula 8 on page 19 just as used on page 10. In this calculation instead of starting with 1/5 it is changed to 16/5 so there is no multiplication by 4 two times required and the same with 4/239. The reason I tried this method was because I saw a calculation of 15 digits and noted that there was only one unit error in the last digit. Due the example I found, I hoped the 40 digits would similar results and they did. By checking the results for 1000 digits there were 7 parts errors in the last digit which effects the last two digits. By checking the results for 2000 digits there were 7 parts errors in the last digit which effects also the last two digits. See the last three page for an expanded listing using this method.

```
(16/5 ) = + 3.2000000000 0000000000 0000000000 0000000000
(16/5 )^ 3/3 = + 0.0426666666 6666666666 6666666666 6666666666
(16/5 )^ 5/5 = + 0.0010240000 0000000000 0000000000 0000000000
(16/5 )^ 7/7 = - 0.0000292571 4285714285 7142857142 8571428571
(16/5 )^ 9/9 = + 0.0000009102 2222222222 2222222222 2222222222
(16/5 )^ 11/11 = - 0.0000000297 8909090909 0909090909 0909090909
(16/5 )^ 13/13 = + 0.0000000010 0824615384 6153846153 8461538461
(16/5 )^ 15/15 = - 0.0000000000 3333333333 3333333333 3333333333
(16/5 )^ 17/17 = + 0.0000000000 0123361882 3529411764 7058823529
(16/5 )^ 19/19 = - 0.0000000000 0004415056 8421052631 5789473684
(16/5 )^ 21/21 = + 0.0000000000 0000159783 0095238095 2380952380
(16/5 )^ 23/23 = - 0.0000000000 0000005835 5533913043 4782608695
(16/5 )^ 25/25 = + 0.0000000000 0000000214 7483648000 0000000000
(16/5 )^ 27/27 = - 0.0000000000 0000000007 9536431407 4074074074
(16/5 )^ 29/29 = + 0.0000000000 0000000000 2962046411 0344827586
(16/5 )^ 31/31 = - 0.0000000000 0000000000 0110837865 7032258064
(16/5 )^ 33/33 = + 0.0000000000 0000000000 0004164816 7718787878
(16/5 )^ 35/35 = - 0.0000000000 0000000000 0000157073 0896822857
(16/5 )^ 37/37 = + 0.0000000000 0000000000 0000005943 3060960864
(16/5 )^ 39/39 = - 0.0000000000 0000000000 0000000225 5408467232
(16/5 )^ 41/41 = + 0.0000000000 0000000000 0000000008 5815541680
(16/5 )^ 43/43 = - 0.0000000000 0000000000 0000000000 3272964845
(16/5 )^ 45/45 = + 0.0000000000 0000000000 0000000000 0125099989
(16/5 )^ 47/47 = - 0.0000000000 0000000000 0000000000 0004791063
(16/5 )^ 49/49 = + 0.0000000000 0000000000 0000000000 0000183820
(16/5 )^ 51/51 = - 0.0000000000 0000000000 0000000000 0000007064
(16/5 )^ 53/53 = + 0.0000000000 0000000000 0000000000 0000000271
(16/5 )^ 55/55 = - 0.0000000000 0000000000 0000000000 0000000010
-----
ATN(16/5) = + 3.1583289575 9809213392 0796243116 6446951613
(4/239 ) = + 0.0167364016 7364016736 4016736401 6736401673
(4/239 )^ 3/3 = - 0.0000000976 6636714833 5214509647 5692049838
(4/239 )^ 5/5 = + 0.0000000000 0102588925 7698589462 8208284435
(4/239 )^ 7/7 = - 0.0000000000 0000001282 8522631138 7788681774
(4/239 )^ 9/9 = + 0.0000000000 0000000000 0174677260 9761567589
(4/239 )^ 11/11 = - 0.0000000000 0000000000 0000002502 0178039686
(4/239 )^ 13/13 = + 0.0000000000 0000000000 0000000000 0370632864
(4/239 )^ 15/15 = - 0.0000000000 0000000000 0000000000 0000005623
-----
ATN(4/239) = + 0.0167363040 0829889545 8152859837 1418109640
-----
ATN(16/5) - ATN(4/239) = PI = + 3.1415926535 8979323846 2643383279 5028841973
-----
THE NUMBER PI TO 35 POSITIONS AFTER THE DECIMAL POINT.
+ 3.1415926535 8979323846 2643383279 50288
```

The total number of number of terms is 36; this is one more than on page 10. The error of the computed value and the correct is 2 parts in the last digit, which effects the last digit. It is interesting to note that 15 digits has 2 part error and 100 digits has 1 part error in the last digit both which effects only the last digit.

The changing error in calculating pi up to 100 digits.

This is standard approach by doing the ATN(1/5) and ATN(1/239) which requires the need for the two multiply by 4, which increases the error in the last digit. Here is a collection of 20 different size of calculation from 5 to 100 in 5 steps increments. The first group is the correct first 100 digits as a sample of what the value should be. The second group has the calculation of pi to five digits with only the first three digits are correct the last two are in error due to a 9 unit error in the last digit. Some calculations are only 1 unit off while 90 units as in 75 digit value. The last three pages use a different method.

The first correct 100 digits.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70679

THE NUMBER PI CALCULATED TO 5 POSITIONS AFTER THE DECIMAL POINT.

3.14168

An error of 9 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159

THE NUMBER PI CALCULATED TO 10 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26544

An error of 9 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535

THE NUMBER PI CALCULATED TO 15 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89812

An error of 19 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793

THE NUMBER PI CALCULATED TO 20 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23860

An error of 14 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846

THE NUMBER PI CALCULATED TO 25 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26428

An error of 5 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433

THE NUMBER PI CALCULATED TO 30 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83280

An error of 1 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279

THE NUMBER PI CALCULATED TO 35 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50304

An error of 16 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288

THE NUMBER PI CALCULATED TO 40 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41988

An error of 17 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971

THE NUMBER PI CALCULATED TO 45 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69432

An error of 33 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399

THE NUMBER PI CALCULATED TO 50 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37496

An error of 14 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

THE NUMBER PI CALCULATED TO 55 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58208

An error of 1 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209

THE NUMBER PI CALCULATED TO 60 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74940

An error of 4 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944

THE NUMBER PI CALCULATED TO 65 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59228

An error of 2 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230

THE NUMBER PI CALCULATED TO 70 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78188

An error of 24 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78164

THE NUMBER PI CALCULATED TO 75 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78164 06376

An error of 90 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78164 06286

THE NUMBER PI CALCULATED TO 80 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78164 06286 20948

An error of 49 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899

THE NUMBER PI CALCULATED TO 85 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86260

An error of 20 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280

THE NUMBER PI CALCULATED TO 90 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34892

An error of 67 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825

THE NUMBER PI CALCULATED TO 95 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34212

An error of 1 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211

THE NUMBER PI CALCULATED TO 100 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70712

An error of 33 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70679

The changing error in calculating pi up to 100 digits.

This is a different approach by doing the $ATN(16/5)$ and $ATN(4/239)$ which eliminates the need for the two multiply by 4, which decreases the error in the last digit. Here is a collection of 20 different size of calculation from 5 to 100 in 5 steps increments. The first group is the correct first 100 digits as a sample of what the value should be. The second group has the calculation of pi to five digits with only the first three digits are correct the last two are in error due to a 2 unit error in the last digit. The samples for 50, 60 and 95 digits have 0 unit's error in the last digit and which effects none of the last digits. Most of the samples have 1 or 2 unit's error and which effects just the last digit. This is an improvement over the above samples on the . See the work on the following web page "cygnus-software.com/misc/pidigits.htm", he did an error by omitting the term $(16/5)^{21/21}$ which would have increased the value by 1 giving a 2 unit error as my sample shows.

The first correct 100 digits.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70679

THE NUMBER PI CALCULATED TO 5 POSITIONS AFTER THE DECIMAL POINT.

3.14161

An error of 2 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159

THE NUMBER PI CALCULATED TO 10 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26538

An error of 3 units in the last digit which which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535

THE NUMBER PI CALCULATED TO 15 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89795

An error of 2 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793

THE NUMBER PI CALCULATED TO 20 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23848

An error of 2 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846

THE NUMBER PI CALCULATED TO 25 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26437

An error of 4 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433

THE NUMBER PI CALCULATED TO 30 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83282

An error of 3 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279

THE NUMBER PI CALCULATED TO 35 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50290

An error of 2 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288

THE NUMBER PI CALCULATED TO 40 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41973

An error of 2 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971

THE NUMBER PI CALCULATED TO 45 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69401

An error of 2 units in the last digit which effects the last 3 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399

THE NUMBER PI CALCULATED TO 50 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

An error of 0 units in the last digit which effects the last 0 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

THE NUMBER PI CALCULATED TO 55 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58211

An error of 2 units in the last digit which effects the last 2 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209

THE NUMBER PI CALCULATED TO 60 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944

An error of 0 units in the last digit which effects the last 0 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944

THE NUMBER PI CALCULATED TO 65 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59231

An error of 1 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230

THE NUMBER PI CALCULATED TO 70 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78162

An error of 2 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78164

THE NUMBER PI CALCULATED TO 75 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78164 06284

An error of 2 units in the last digit which effects the last 1 digit.

The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510

58209 74944 59230 78164 06286

THE NUMBER PI CALCULATED TO 80 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20902

An error of 3 units in the last digit which effects the last 3 digit.
The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899

THE NUMBER PI CALCULATED TO 85 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86281

An error of 1 units in the last digit which effects the last 1 digit.
The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280

THE NUMBER PI CALCULATED TO 90 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34826

An error of 1 units in the last digit which effects the last 1 digit.
The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825

THE NUMBER PI CALCULATED TO 95 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211

An error of 0 units in the last digit which effects the last 0 digit.
The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211

THE NUMBER PI CALCULATED TO 100 POSITIONS AFTER THE DECIMAL POINT.

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70681

An error of 2 units in the last digit which effects the last 2 digit.
The correct value of the number PI is

3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70679