

How to calculate PI to 40 digits

Here is how to calculate PI by yourself using only paper and pencil. One of the most used formula is $PI = 4 * (4 * ATN(1/5) - ATN(1/239))$, the function $ATN(X) = X - X^3/3 + X^5/5 - X^7/7 + X^9/9$ etc. can be used to calculate the ATN function. The process continues until the value reaches enough 0's and no longer affects the total sum. See the bottom of this page how it works to produce the values needed.

The challenge is to do a small sample of what William Shanks did, but on a much larger scale. To do it correctly all the work must be done with paper and pencil only, no calculating type device, a calculator or computer even an abacus. This would require you to know how to do long division without any aid other than your own mind plus paper and pencil. Can you do the challenge or will you just say it is too hard I am not capable to do the task, which are you. Without looking at the following page except to see if you can do the first two terms correct. The work can be done in several days if you work hard and steady and long, or use a group effort to speed up the work. There are some short cuts starting with doing the powers of 1/5, after the first term divide by 25 from sub term to sub term. In this case multiply by 4 and divide by 100 which are the same as shifting to the right by two digits. Then do the division by the term number i.e. 3, 5, 7 etc. Remember to alternate the addition and subtraction of every other term. Have fun with this problem. These formulas are sometime referred to as Machin-like formula. There are many longer formulas that take more and some less total terms to reach the same number of digits.

On the last 16 pages I have included my work on calculating the required 40 digits. I produced the work sheet in EXCEL to make keeping the columns of digits in order; it is hard enough to keep 5 digits straight, let alone 40. Have extra work sheets as you may have to start a calculation over. Try and do as much of the work without looking at my work. For more of my work on PI go to <http://engert.us/erwin/Miscellaneous.html> where you can find this and other files.

Here is a term by term breakdown for the solution of computing PI to 35 digits. Note how each term is getting closer to 0. You must always calculate more digits than required to for the accuracy you desire hence 40 digits to get a 35 digit answer. The reason I picked 35 digits was that Ludolph Van Ceulen (1540 - 1610) spent most of his life working out PI to 35 decimal places. PI is sometimes known as Ludolph's Constant the value is carved on his tombstone.

(1/5)=	+	0.2000000000	0000000000	0000000000	0000000000
(1/5)^ 3/3=	-	0.0026666666	6666666666	6666666666	6666666666
(1/5)^ 5/5=	+	0.0000640000	0000000000	0000000000	0000000000
(1/5)^ 7/7=	-	0.0000018285	7142857142	8571428571	4285714285
(1/5)^ 9/9=	+	0.0000000568	8888888888	8888888888	8888888888
(1/5)^ 11/11=	-	0.0000000018	6181818181	8181818181	8181818181
(1/5)^ 13/13=	+	0.0000000000	6301538461	5384615384	6153846153
(1/5)^ 15/15=	-	0.0000000000	0218453333	3333333333	3333333333
(1/5)^ 17/17=	+	0.0000000000	0007710117	6470588235	2941176470
(1/5)^ 19/19=	-	0.0000000000	0000275941	0526315789	4736842105
(1/5)^ 21/21=	+	0.0000000000	0000009986	4380952380	9523809523
(1/5)^ 23/23=	-	0.0000000000	0000000364	7220869565	2173913043

(1/5)^ 25/25=	+	0.0000000000	0000000013	4217728000	0000000000
(1/5)^ 27/27=	-	0.0000000000	0000000000	4971026962	9629629629
(1/5)^ 29/29=	+	0.0000000000	0000000000	0185127900	6896551724
(1/5)^ 31/31=	-	0.0000000000	0000000000	0006927366	6064516129
(1/5)^ 33/33=	+	0.0000000000	0000000000	0000260301	0482424242
(1/5)^ 35/35=	-	0.0000000000	0000000000	0000009817	0681051428
(1/5)^ 37/37=	+	0.0000000000	0000000000	0000000371	4566310054
(1/5)^ 39/39=	-	0.0000000000	0000000000	0000000014	0963029202
(1/5)^ 41/41=	+	0.0000000000	0000000000	0000000000	5363471355
(1/5)^ 43/43=	-	0.0000000000	0000000000	0000000000	0204560302
(1/5)^ 45/45=	+	0.0000000000	0000000000	0000000000	0007818749
(1/5)^ 47/47=	-	0.0000000000	0000000000	0000000000	0000299441
(1/5)^ 49/49=	+	0.0000000000	0000000000	0000000000	0000011488
(1/5)^ 51/51=	-	0.0000000000	0000000000	0000000000	0000000441
(1/5)^ 53/53=	+	0.0000000000	0000000000	0000000000	0000000016

ATN(1/5)=	+	0.1973955598	4988075837	0049765194	7902934477
4*ATN(1/5)=	+	0.7895822393	9952303348	0199060779	1611737908

(1/239)=	+	0.0041841004	1841004184	1004184100	4184100418
(1/239)^ 3/3=	-	0.0000000244	1659178708	3803627411	8923012459
(1/239)^ 5/5=	+	0.0000000000	0025647231	4424647365	7052071108
(1/239)^ 7/7=	-	0.0000000000	0000000320	7130657784	6947170443
(1/239)^ 9/9=	+	0.0000000000	0000000000	0043669315	2440391897
(1/239)^ 11/11=	-	0.0000000000	0000000000	0000000625	5044509921
(1/239)^ 13/13=	+	0.0000000000	0000000000	0000000000	0092658216
(1/239)^ 15/15=	-	0.0000000000	0000000000	0000000000	0000001405

ATN(1/239)=	+	0.0041840760	0207472386	4538214959	2854527411
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4*ATN(1/5)-ATN(1/239)=PI/4=	+	0.7853981633	9744830961	5660845819	8757210497
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4*(4*ATN(1/5)-ATN(1/239))=PI=	+	3.1415926535	8979323846	2643383279	5028841988
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The number PI correct to 40 positions after the decimal point is.
+ 3.1415926535 8979323846 2643383279 5028841971

The total number of number of terms is 35. The error of the computed value and the correct is 17 parts in the last digit, affecting the last two digits.

1/5 terms Neg + Positive

Carry	00000	10122	12232	32323	23	242	65544	46564	6445
3	00266	66666	66666	66666	66	666	66666	66666	66666
7	00000	18285	71428	57142	85	714	28571	42857	14285
11		00018	61818	18181	81	818	18181	81818	18181
15			02184	53333	33	333	33333	33333	33333
19			00002	25941	05	263	15789	47368	42105
23				00364	72	208	69565	21739	13043
27					49	710	26926	96296	29629
31					60	069	27366	60645	16129
35							09817	06810	51428
39							00014	09630	29202
43								02045	60302
47								00002	99441
51									00441
total	00266	84971	02100	71630	94783	96268	69213	74185	
carry	00000	00001	10121	22222	24243	4	3324	45454	5455
1	20000								
5	00006	40000							
9		00568	88888	88888	88	888	88888	88888	88888
13			63015	38461	53	846	15384	61538	46153
17			00077	10117	64	705	88235	29411	76470
21				09986	43	809	52380	95238	09523
25				00013	42	177	28000	00000	00000
29					01	851	27900	68969	51724
33					00	002	60301	04824	24242
37							00371	45663	10054
41								53634	11345
45								00078	19749
49									11488
53									00016
total	20006	40869	51981	47467	95281	61463	48243	08694	
Sub	00266	84971	02100	71630	94783	96268	69213	74185	
ATN	19739	55598	49880	75837	00487	65194	79029	34477	
X4	78958	22393	99523	03348	01990	60779	16117	37908	

1/239

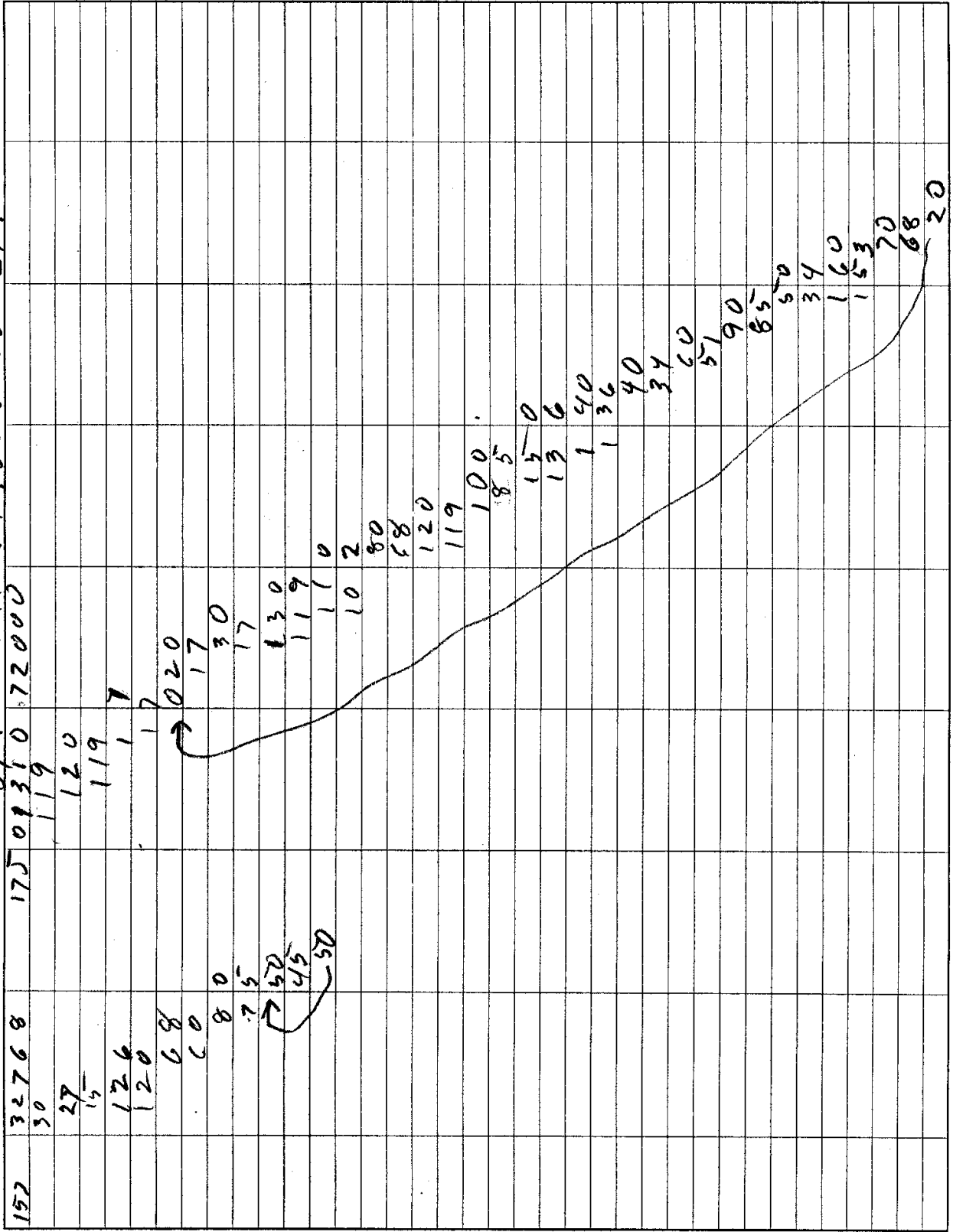
1	00418	41004	18410	04184	10041	84100	41841	00418
3		00732	49775	36125	14108	82235	67696	37378
5			01282	36157	21232	36828	52603	55584
7				02244	99146	04492	86361	93105
9					03930	23837	19635	27077
11						06880	54896	07137
13							12045	56811
15								21087
Carry	00000	00000	00000	01001	00010	11112	11100	1211
3		00244	16591	28208	38036	27411	89230	12459
7				00320	71306	57784	69471	70443
11						00625	50445	09921
15								01405
Total		06244	16591	79029	09342	85822	09146	94228
Carry	00000	00000	00000	10100	00112	20011	02012	1112
1	00418	41004	18410	04184	10041	84100	41841	00418
5			00256	47231	44246	47365	70520	71108
9					00436	69315	24403	91897
13							00926	58216
Total	00418	71004	18666	51415	54725	00781	37692	31639
Sub	00000	00244	16591	79029	09342	85822	09146	94228
ATN ²⁵⁹	00418	40760	02074	72386	45982	14959	28545	27411
4	ATN(4)	78958	22393	99523	03348	60779	16117	37908
	ATN(2)	00418	40760	02074	45382	14959	28545	27411
	Total	78539	81633	97448	30961	45819	87572	10497
X4								
74	3014159	26535	89793	23846	26473	83279	50288	41988

Powers of $\frac{1}{5}$

1	2.0000								
3	008000								
5	00032								
7	00001	280000							
9	05120								
11	00204	80000							
13	00008	192000	00000						
15		32768	00000						
17		01310	20000						
19		00052	42880						
21		2	09715		20000				
23			08388		60800				
25			00335		54432				
27			00013		42177	28000			
29					53687	09120			
31					02147	48364	80000		
33					00085	89934	59200		
35					00003	43597	38368	00000	
37						13743	89534	72000	
39						00549	75581	38880	
41						00021	99023	25555	
43							87960	93022	
45							03518	43720	
47							00140	73748	
49							00005	62948	
51								22516	
53								900	

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195	00052	42880	00000	00000	00000	00000	00000
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times	00388	608 00	00000	00000	00000	00000	00000	00000	00000	00000
1	69									
2	148									
3	238									
4	108									
5	92									
6	166									
7	161									
8	50									
9	46									
10	48									
11	46									
12	200									
13	184									
14	115									
15	138									
16	161									
17	184									
18	207									
19	230 OK									
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---	00002	89934	59200	00000					
335	00085	8							
	66	8							
	19	8			098	17	06810	5	1428
	19	8		35	00003	435	97	38368	00000
	099	0			3	15			
	099	0				285			
	034	33				280			
	33	1				59			
	1	1				35			
	272					247			
	264					245			
	80					2	38		
	66					2	10		
	140						283		
	132						280		
	80						36		
	---	00371					35		
	375	13743							
	411								
	264								
	259								
	53								
	37								
	168								
	148								
	1209								
	1								
	85								
	245								
	222								
	233								
	222								
	114								
	111								
	37								
	37								
	OK								
	1 35	1							
	2 70	2							
	3 105	3							
	4 140	4							
	5 175	5							
	6 210	6							
	7 245	7							
	8 280	8							
	9 315	9							
	10 350	10							
	OK	OK							
	0200								
	185								
	150								
	148								
	100								
	70								
	300								
	280								
	20								

39 41 43 45

----- 00014 09630 29202 ----- 415 00021 99023 25555

395	00549	75581	38880	415	00021	99023	25555
	39				20	5	
	159				1	49	
	156				1	23	
	3	76				260	
	3	51				276	
		245				142	
		234				123	
		118				193	
		117				164	
		1	3			292	
		7	8			287	
		3	58			55	
		3	51			41	
			78			145	
			78			123	
			080			225	
	-----	02045	60302			205	
	435	87960	93022				
		86			-----	00078	1
		196			75)	03518	43720
		172				315	
		240				368	
		215				360	
		259				84	
		258				45	
1	39	41	43		45		
2	78	82	86		90		
3	117	123	129		135		
4	156	164	172		180		
5	195	205	215		225		
6	234	246	258		270		
7	273	287	301		315		
8	312	328	344		360		
9	351	369	387		405		
10	390	410	430		450		

47 9951 53

2 99441 73748 49 0000 11488 51 22517

475	00140	73748	49	0000	11488	51	22517
	94			4	62949		204
	467				72		211
	423				49		204
	443				239		77
	423				196		
	207				434		
	188				392		
	194				429		
1	47	49					
2	94	98					
3	141	147					
4	188	196	00016				
5	235	245	00800		00418	41	
6	282	294	53		2392100000	00000	00000
7	329	343	370		952		
8	376	392	318		440		
9	423	441	52		239		
10	470	490			2010		
					1912		
					980		
					956		
					239		
1	57121		239				
2	114242		X 239				
3	171362		2157				
4	228484		717				
5	342726		478				
6	399847		57121				
7	456968						
8	514089						
9	5714089						
10	571210						

†3 244 16591 38708 38036 22411 89230 12459
 0000 00732 49775 36125 14108 82235 67690 37578
 571215 00418 41004 18410 04184 10041 84100 41841 00418

399	847	9	1277	04					
18	5630		1142	42					
17	13633		134	621					
1	42674		114	242					
1	14242		20	2790					
	28432	1	17	1363					
	22848	4	3	24270					
	5588	78	2	85605					
	5140	89		386657					
	442	894		342726					
	399	847		439281					
	43	0471		399847					
	34	9847		394348					
	3	06240		342726					
	2	85605		41	6224				
		20635	0	51	4089				
		171363	74	2135	0				
		3498	26	17136	30				
		3427	481	4214	47				
		71	52	3998	334				
		52	121	216	363				
		14	3608	174	8711				
		11	4242	349	9847				
		12	93664	34	88648				
		2	85605	4	56968				
			80591		31680				
			57121						
			234700						
			228484						
			62160						
			57121						
			5	03941					
			9	56968					
			4	46973					
			45	6968					
			1277	70					

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 00000 02282 36157 21232 36828 52603 55544

971215	00282	36157	21232	36828	52603	55544
1142	42					
139	941					
114	242					
25	6995					
22	8484					
2	85117					
2	28484					
	56633	2				
	51408	9				
	5224	31				
	5140	89				
	83	422				
	57	121				
	26	3013				
	22	8484				
	3	45292				
	3	42786				
		256636				
		228484				
		281828				
		228484				
		530442				
		514089				
		1				
		63538				
		14242				
		492965				
		456968				
		359972				
		342726				
		172466				
		171363				
		110303				
		97121				
		53182				

531825
 514089
 177365
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 57121
 290444
 255605
 4839

9 11 9 00000 00436 69315 24403 91897
 -- 00000 03930 32837 19635 27077

57215	02244	99146	04492	86301	93108				
	1713	63							
	531	361	---	00000	06880	54896	09137		
	514	089	57215	03930	23837	19635	27077		
	17	2724		3427	26				
	17	1363		502	978	00625	50445	09921	
		13616	0	456	968	006880	54896	09137	
		11424	2	46	0103	668			
		2191	84	45	6968	26			
		1713	63		31357	60			
		478	214		28560	55			
		456	968		2796	55			
		21	2469		2284	69			
		17	1363		511	84			
		4	11062		456	968			
		3	99847		54	8883			09
			11215	8	51	4089			99
			5712	1	3	47945			101
			5503	76	3	42726			99
			5140	89					23
			362	873					22
			342	726					17
			20	1470					16
			17	1363					
			3	01071					
			2	85605					
				15466	9				
				11424	2				
				4042	73				
				3998	47				
				44	3610				
				3	9847				
				4	37638				
				3	99847				
					37791				

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----- 00000 12045 56811 ----- 571215 00000 21087

571215	06880	54896	09135		571215	12045	56811
	5212	1				11424	2
	1168	44				621	36
	1142	42				571	21
	26	0289				50	1581
	72	8484				45	6968
1	13	3	18056		1	15	
2	26	2	85605		2	30	
3	39	3	32451	0	3	45	
4	52	2	28560	5	4	60	
5	65	3	38905	9	5	75	
6	78	3	34272	6	6	90	
7	91	4	46333	1	7	105	
8	104	4	45696	8	8	120	
9	117	5	63633	3	9	135	
0	130	5	7121	1	10	150	
			65125				
			8004				
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13	00000	926	58216			00000	01405
		12045	56811			00000	21087
		1174				15	15
		36				60	60
		26				60	60
		87					87
		78					75
		75					12
		65					
		1	06				
		1	04				
			28				
			26				
			21				
			13				
			81				
			78				
			3				